Exemple p.127

$$A = \begin{pmatrix} 2 & 1 & -2 \\ -3 & -1 & 4 \\ -2 & 3 & 2 \\ 0 & 4 & 0 \end{pmatrix} \in M_{4\times 3}(\mathbb{R})$$

$$A \sim \begin{pmatrix} 2 & 0 & -2 \\ 0 & 1 & 0 \\ -3 & 0 & 4 \\ -2 & 0 & 2 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & -2 \\ 0 & 0 & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \sim \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}}_{A_{ER}}$$

- $\operatorname{Im}(A) = \operatorname{Span}\left\{ \begin{pmatrix} 2 \\ -3 \\ -2 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 3 \\ 4 \end{pmatrix}, \begin{pmatrix} -2 \\ 4 \\ 2 \\ 0 \end{pmatrix} \right\}$ et les colonnes de A sont lin. indép. Donc $\operatorname{rang}(A) = 3$.

- $\operatorname{Im}(A^T) = \operatorname{Span}\left\{ \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}, \begin{pmatrix} -3 \\ -1 \\ 4 \end{pmatrix}, \begin{pmatrix} -2 \\ 3 \\ 2 \end{pmatrix}, \begin{pmatrix} 0 \\ 4 \\ 0 \end{pmatrix} \right\}$

$$= \operatorname{Span}\left\{ \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}, \begin{pmatrix} 0 \\ 4 \\ 0 \end{pmatrix}, \begin{pmatrix} -3 \\ -1 \\ 4 \end{pmatrix} \right\}$$

donc $\dim \operatorname{Im}(A^T) = 3$.

- $\operatorname{Ker}(A) = \{\vec{0}\}$, $\dim \operatorname{Ker}(A) = 0$.

Th. du rang: $3 + 0 = 3$ ok !

- $\operatorname{Ker}(A^T)$: $\left(\begin{array}{cccc|c} 2 & -3 & -2 & 0 & 0 \\ 1 & -1 & 3 & 4 & 0 \\ -2 & 4 & 2 & 0 & 0 \end{array}\right)$

$$\sim \left(\begin{array}{cccc|c} 1 & -1 & 3 & 4 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 2 & 8 & 8 & 0 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & 0 & 3 & 4 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \end{array}\right)$$

$$\sim \left(\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \end{array}\right) \text{ d'où } \begin{cases} x_1 + x_4 = 0 \\ x_2 = 0 \\ x_3 + x_4 = 0 \end{cases} \quad x_4 \text{ libre}$$

$\Rightarrow \ker(A^T) = \text{span}\left\{ \begin{pmatrix} -1 \\ 0 \\ -1 \\ 1 \end{pmatrix} \right\}$

et $\dim \ker(A^T) = 1$

Th. du rang : $3 + 1 = 4$ ok !

<u>Remarque:</u> On observe que $\text{Im}(A) \neq \text{Im}(A_{ER})$

$$(A_{ER})^T = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$$(A^T)_{ER} = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$$

et $\text{Lgn}\left((A^T)_{ER}\right) \neq \text{Lgn}\left((A_{ER})^T\right)$